Answers to "Test Yourself" No. 7

Design the Race Track With Solution

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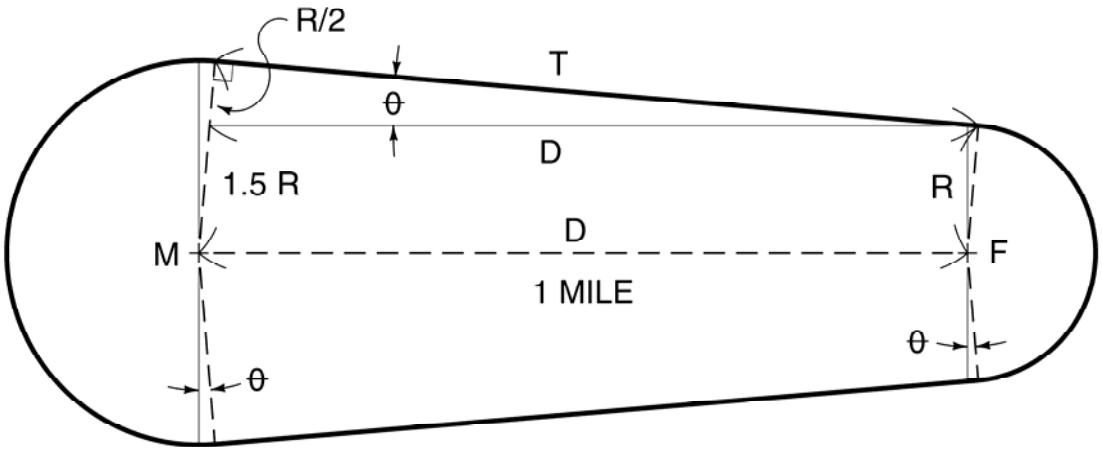
For the solution below, refer to this sketch:



NOT TO SCALE

$$\theta(radians) = \sin^{-1}\left[\left(\frac{R}{2}\right)/D\right] = \sin^{-1}\frac{R}{2D}$$

$$T = \left[D^2 - \left(\frac{R}{2}\right)^2\right]^{0.5}$$

$$Perimeter\ (P) = 1.5R(\pi + 2\theta) + R(\pi - 2\theta) + 2T$$

$$P = 1.5R\pi + 3R\theta + R\pi - 2R\theta + 2T$$

$$P = R\left(\frac{3}{2}\pi + 3\theta + \pi - 2\theta\right) + 2T$$

$$P = R\left(\frac{5}{2}\pi + \theta\right) + 2T$$

$$P = R\left(\frac{5}{2}\pi + \theta\right) + 2\left[D^2 - \left(\frac{R}{2}\right)^2\right]^{0.5}$$

$$P = R\left(\frac{5}{2}\pi + \sin^{-1}\frac{R}{2D}\right) + 2\left[D^2 - \left(\frac{R}{2}\right)^2\right]^{0.5}$$

For our knowns: *P = 3.42 miles* *D = 1.0 miles*

This is a transcendental equation and will require an iterative solution for R. When this is accomplished, R = 0.17977 miles.

Now for the area (3 in radians):

$$Area = \frac{\pi}{2}\left(\frac{3}{2}R\right)^2 + \left(\frac{3}{2}R\right)^2\theta + \frac{\pi}{2}R^2 - R^2\theta + T\left(\frac{3}{2}R + R\right)$$

$$Area = R^2\left(\frac{13}{8}\pi + \frac{5}{4}\theta\right) + T\frac{5}{2}R$$

For our knowns: *D = 1.0 mile* *R = 0.17977 miles*

$$\theta = \sin^{-1}\frac{R}{2D} = 0.090006477 \text{ radians}$$

$$T = \left[D^2 - \left(\frac{R}{2}\right)^2\right]^{0.5} = 0.995952151 \text{ miles}$$

$$Area = 0.616224175 \text{ miles}^2$$

$$Area = 394.39 \text{ acres}$$